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Surface light backscattering from semi-infinite disordered magnetic systems and applications to Fe_8B_2

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Abstract. Calculations are presented for the differential cross section of light backscattered from surfaces of disordered magnetic systems, in the site disorder decoupling approximation. The results are general and show the distinct contributions to scattering owing to the presence of the surface and of the magnetic disorder, for the system one-excitations. They are applied for scattering from surfaces of amorphous magnets at small frequency shifts in the hydrodynamic region of scattering from small wavevector excitations. Although this 'washes out' the site disorder the results are non-trivial and illustrate scattering from diffusive and magnon-like bulk modes, as in Fe_8B_2, where good agreement with experimental measurements is obtained.

1. Introduction

Techniques of light backscattering by surfaces have been used experimentally to probe excitations in disordered magnetic systems, as in the ferromagnetic metallic glasses (Chang *et al* 1978, Malozemoff *et al* 1979a, b), dilute magnetic semiconductors (Petrou *et al* 1983), and insulating spin glasses (Zirngiebl *et al* 1984).

There is no general theoretical treatment of light backscattering from surfaces of disordered magnetic systems, and the purpose of this paper is to present such a formal calculation. The results obtained are general, and are illustrated at small frequency shifts for backscattering in surfaces of amorphous magnets, in the hydrodynamic region of small wavevector excitations, and are also compared to experimental measurements in Fe_8B_2.

2. Theory

The disordered magnetic system (medium 1), and free space (medium 2), are considered to occupy the half-spaces $z < 0$ and $z > 0$, respectively. Light incident from free space into the material is characterised by an electric field vector $\boldsymbol{E}_1$. The polarisation induced

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in the system is linear to first order in the local spin operators, and can be written in the laboratory frame following Khater and Continentinho (1982), in the form

$$P^n_{\mathrm{ind}} = \sum_i \sum_{\alpha\beta} \sum_m \tilde{\mathbf{A}}^{n\beta}(\boldsymbol{r}_i)\chi^\beta_\alpha(\boldsymbol{r}_i)\mathbf{A}^{\alpha m}(\boldsymbol{r}_i) f^m E^m_{\mathrm{I}} \tag{1}$$

where $n, m = x, y, z$ denote laboratory coordinates, and $\alpha, \beta = x', y', z'$ denote coordinates in the local ionic frames such that z' is parallel to the direction of the spin on each site. $\mathbf{A}^{\alpha m}(\boldsymbol{r}_i)$ are transformation matrices that convert the components of the incident electric field vectors at a spin site $\boldsymbol{r}_i$ from laboratory to ionic coordinates, and the matrices $\tilde{\mathbf{A}}^{n\beta}(\boldsymbol{r}_i)$ convert the scattered components in the opposite sense. The quantities f^m are Fresnel coefficients for optical transmission at the interface $z = 0$, at the incident radiation frequency. $\chi^\beta_\alpha(\boldsymbol{r}_i)$ is the polarisability tensor, which to first order is linear in the spin operators of the magnetic system.

The cross section for light backscattered from a surface A' of a semi-infinite solid at an angle θ_{S}, and at a frequency shift of

$$\pm\omega = \omega_{\mathrm{I}} - \omega_{\mathrm{S}} \tag{2}$$

for Stokes and anti-Stokes, respectively, can be expressed for the disordered system in the form of a configurational average (Khater and Continentinho 1982):

$$\begin{aligned}\langle\sigma\rangle = A'F(n(\omega)+{\textstyle\frac{1}{0}}) \sum_l \sum_{kk'} \sum_{\substack{\alpha\beta\\ \alpha'\beta'}} \sum_{\substack{nm\\ n'm'}} & (e^l_{\mathrm{S}})^* e^{l}_{\mathrm{S}} (\mathbf{g}^{ln})^* \mathbf{g}^{l'n'} (f^m)^* f^{m'} \\ & \times (E^m_{\mathrm{I}})^* E^{m'}_{\mathrm{I}} ((B_{\mathrm{L}} + k_z)^{-1})^* (B_{\mathrm{L}} + k'_z)^{-1} \\ & \times \langle \varphi^{n\beta}_{\alpha m}(\boldsymbol{k}) \varphi^{n'\beta'}_{\alpha' m'}(\boldsymbol{k}') \,\mathrm{Im}\, G(\chi^\beta_\alpha(\boldsymbol{k})^*; \chi^{\beta'}_{\alpha'}(\boldsymbol{k}'))_\omega \rangle. \end{aligned} \tag{3}$$

$\boldsymbol{e}_{\mathrm{I}}$ and $\boldsymbol{e}_{\mathrm{S}}$ are the polarisation unit vectors for incident and scattered light. F is a product of constant terms. $\boldsymbol{k}$ and $\boldsymbol{k}'$ characterise the wavevector of magnetic excitations in the disordered system. The quantities $\mathbf{g}$ are transmission matrices for light scattered out of the magnetic material at the interface into free space (Bennett *et al* 1972). The scattered light is incoherent.

The site disorder, that is the random variation of the matrix elements of $\mathbf{A}^{\alpha m}(\boldsymbol{r}_i)$ with position vector $\boldsymbol{r}_i$, leads to a new structure factor determined from the quantities

$$\varphi^{n\beta}_{\alpha m}(\boldsymbol{k}) = \frac{1}{(2\pi)^{1/2} N} \sum_i \tilde{\mathbf{A}}^{n\beta}(\boldsymbol{r}_i)\mathbf{A}^{\alpha m}(\boldsymbol{r}_i) \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_i). \tag{4}$$

The effect of the surface is to break the symmetry along the z direction, introducing a geometrical structure factor (Cottam and Lockwood 1986), given for backscattering from the semi-infinite medium in the form

$$1/(B_{\mathrm{L}} + k_z) = 1/(k^{2\mathrm{I}}_z + k^{2\mathrm{S}}_z + k_z) \tag{5}$$

$k^{2\mathrm{I}}_z$, $k^{2\mathrm{S}}_z$ being the z components of incident and scattered radiation in free space.

The form of equation (3) couples the site disorder to the dynamics of the magnetic excitations in the disordered system under the double summation $\boldsymbol{kk}'$. A useful approximation when taking the configurational average is to decouple the site disorder from the excitation dynamics (Bhattacharjee *et al* 1978), which restores translational invariance parallel to the solid surface, such that $\boldsymbol{k}_1$ satisfies wavevector conservation. This is a reasonable approximation for light scattering, as the wavelengths of the incident and scattered light are much greater than the characteristic dimensions of the site disorder in the material. For incident light polarised in the yz plane, and for light backscattered at an angle θ_S to the perpendicular to the surface, the site disorder decoupling approximation leads from equation (3) to the following contribution to the differential cross section, arising from the g^{xx} term:

$$\begin{aligned}\frac{d^2\langle\sigma\rangle}{d\Omega\, d\omega} = A'F'|g^{xx}|^2(n(\omega)+\tfrac{1}{0}) \sum_{\alpha\alpha'} \sum_{k_{\|},k_z} |B_L + k_z|^{-2} \\ \times (+\sin^2\theta_S |f^z|^2 \langle \varphi_{\alpha z}^{x\alpha}(\boldsymbol{k})^* \varphi_{\alpha' z}^{x\alpha'}(\boldsymbol{k})\rangle \\ - \sin\theta_S \cos\theta_S (f^z)^* f^y \langle \varphi_{\alpha z}^{x\alpha}(\boldsymbol{k})^* \varphi_{\alpha' y}^{x\alpha'}(\boldsymbol{k})\rangle \\ - \sin\theta_S \cos\theta_S f^z (f^y)^* \langle \varphi_{\alpha' y}^{x\alpha'}(\boldsymbol{k})^* \varphi_{\alpha z}^{x\alpha}(\boldsymbol{k})\rangle \\ + \cos^2\theta_S |f^y|^2 \langle \varphi_{\alpha y}^{x\alpha}(\boldsymbol{k})^* \varphi_{\alpha' y}^{x\alpha'}(\boldsymbol{k})\rangle) \\ \times \langle \mathrm{Im}\, G(S_k^{\alpha}; S_k^{\alpha'})_\omega\rangle. \end{aligned} \qquad (6)$$

There are additional contributions, arising from the other g-terms, for the same configuration of incident and scattered light. F' is a product of constant terms. G is the Green function over the polarisability operators of the system. Since P of equation (1) is linear in the spin operators, only one superscript is retained for the spin coordinates in the local ionic frame.

The central result of this paper is exemplified by equation (6). Similar expressions can be obtained for scattered light arising due to the products of the other g-terms, and also collected under a different experimental configuration. It is general, and gives the one-excitation differential cross section for light backscattering from the surface of a disordered magnetic system. It involves a summation over all the wavevectors, in contrast to scattering from surfaces of ordered magnetic systems to which only the summation over the structure factor of equation (5) in k_z contributes. It shows, further, the distinct contributions to scattering owing to the disorder and to surface effects, for all the system magnetic one-excitations.

3. Applications to amorphous magnets

The general result obtained in § 2 is applied next to scattering from the surface of metallic glasses (Chang *et al* 1978, Malozemoff *et al* 1979a, b), which are examples of an amorphous ferromagnet (Mook 1978, Murani 1978, Halperin and Saslow 1977, Dzyaloshinskii and Volovik 1978, Schwabl and Michel 1970; Continentinho and Rivier 1979a, b). We consider the domain of relatively small Stokes and anti-Stokes frequency shifts of backscattering in the hydrodynamic region from small wavevector excitations. In this region surface backscattering probes bulk excitations. Although this limit washes out the magnetic structure contributions due to the site disorder, it is nevertheless non-trivial to consider this in order to illustrate the method, and we find good agreement with experimental results in Fe_8B_2.

The dynamics of bulk excitations in amorphous magnets has been intensively investigated experimentally and theoretically (Halperin and Saslow 1977, Dzyaloshinski and Volovik 1978, Continentinho and Rivier 1979a, b). There is evidence in these systems for two types of bulk excitations, diffusive and magnon-like modes, corresponding in our formalism to

$$\alpha, \alpha' = z, z' \qquad \alpha, \alpha' = +, - \tag{7}$$

respectively. For small-wavevector excitations $\boldsymbol{k} \simeq 0$, and near-perpendicular backscattering when $\theta_S \simeq 0$, the differential cross section from bulk diffusive modes at room temperature can be expressed, using equation (6), in the form

$$\frac{\mathrm{d}^2\langle\sigma_{zz}\rangle}{\mathrm{d}\Omega\,\mathrm{d}\omega} \simeq \frac{k_B T}{\hbar\omega}\sum_{k_z}\left|\frac{1}{B_L + k_z}\right|^2 \langle \mathrm{Im}\, G(S_k^z; S_k^z)_\omega\rangle. \tag{8}$$

Equally, for backscattering from bulk magnon-like modes

$$\frac{\mathrm{d}^2\langle\sigma_{+-}\rangle}{\mathrm{d}\Omega\,\mathrm{d}\omega} \simeq \frac{k_B T}{\hbar\omega}\sum_{k_z}\left|\frac{1}{B_L + k_z}\right|^2 \langle \mathrm{Im}\, G(S_k^+; S_k^-)_\omega\rangle. \tag{9}$$

Equations (8) and (9) show no structure contributions due to magnetic disorder, and consequently are independent of the product terms in $\mathbf{g}$. They contain only the structure effects owing to the presence of the surface. The disorder however is still contained in the dynamics of the magnetic excitations in the disordered magnetic system, in the configurational averages of the Green functions.

These averages for the decoupled diffusive and magnon-like modes have been calculated for the isotropic amorphous ferromagnet assuming negligible anisotropy (Continentinho and Rivier 1979a, b). This magnetic system is of interest here for illustrating the formalism by the case of Fe_8B_2 as an example. The configurational averages are given by the following expressions:

$$\langle \mathrm{Im}\, G(S_k^z; S_k^z)_\omega\rangle = \chi_0 \omega b k^2/(\omega^2 + b^2k^4) \tag{10}$$

and

$$\langle \mathrm{Im}\, G(S_k^+; S_k^-)_\omega\rangle = (2/\pi N)M(T)\Gamma(\boldsymbol{k})/[(\omega - \omega_0(\boldsymbol{k}))^2 + \Gamma^2(\boldsymbol{k})] \tag{11}$$

where b is a constant, $M(T)$ the magnetisation of the amorphous ferromagnet at temperature T, $\omega_0(\boldsymbol{k})$ the dispersion curve of the magnon-like mode and $\Gamma(\boldsymbol{k})$ is the inverse of the lifetime for the mode.

3.1. Diffusive modes

The differential cross section for backscattering from diffusive modes in an isotropic amorphous ferromagnet will now be calculated in the hydrodynamic region and for near-perpendicular θ_S, using equations (8) and (10). If x is the dimensionless variable defined as

$$x = (1/2\pi)\omega/b|B_L|^2 \tag{12}$$

the differential cross section at room temperature is obtained in the form

$$\mathrm{d}^2\langle\sigma_{zz}\rangle/\mathrm{d}\Omega\,\mathrm{d}x \simeq TM(T)(1.75 + x)/[x^{1/2}(9 + x^2)]. \tag{13}$$

The result of (13) is plotted in figure 1, which shows that bulk diffusive modes give rise to backscattering as mainly a central peak around $\omega = 0$ with a tail that extends to higher frequencies, decaying slowly following a power law, rather than exponentially.

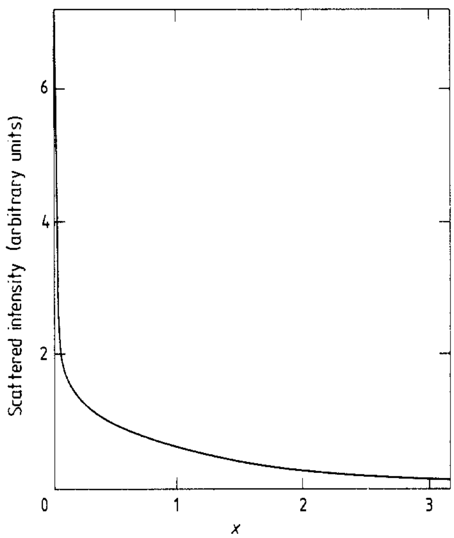

Figure 1. The theoretical differential cross section for near-perpendicular surface backscattering from bulk diffusive modes in the amorphous ferromagnet Fe_8B_2, plotted as a function of the dimensionless variable $x = (1/2\pi)\omega/b|B_L|^2$. See § 4.1 for details.

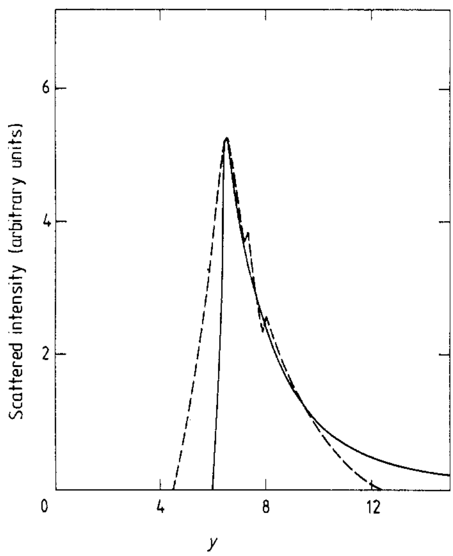

Figure 2. The differential cross section for near-perpendicular surface backscattering from bulk magnon-like modes in the amorhous ferromagnet Fe_8B_2, plotted as a function of the dimensionless variable $y = \omega/\gamma D|B_L|^2$. See § 4.2 for details. Full curve, theoretical result; broken curve, experimental results for comparison (after Chang *et al* 1978).

3.2. *Magnon-like modes*

The differential cross section for backscattering from bulk magnon-like modes is calculated in the hydrodynamic region for near perpendicular incidence using equations (9) and (11). In this limit we approximate the inverse of the lifetime by a constant value Γ_0; the dispersion curve for this mode is also well known, namely

$$\omega_0(\boldsymbol{k}) = \gamma(H(H + 4\pi M))^{1/2} + \gamma D k^2 \tag{14}$$

where H is the externally applied magnetic field, γ the gyromagnetic ratio, and D the stiffness constant. Let y be the dimensionless variable defined as

$$y = \omega/\gamma D|B_L|^2.$$

From the experimental data of Chang *et al* (1978) and Malozemoff *et al* (1979a, b), the constant values can be expressed in the form $\Gamma_0^{-1}\gamma D|B_L|^2 = 2\pi$ and $(H(H + 4\pi M))^{1/2}/D|B_L|^2 = 6$ for fields of the order of 1 kG. Substituting for $\omega_0(\boldsymbol{k})$ from equations (14) into (11), the cross section at room temperature is obtained in the form:

$$\frac{\mathrm{d}^2\langle\sigma_{+-}\rangle}{\mathrm{d}\Omega\,\mathrm{d}y} \simeq TM(T)\frac{1}{y}\left(-\frac{\tan^{-1}|y-6|}{(y-5)^2+1} + \frac{C}{2((y-5)^2+1)((y-6)^2+1)^{1/4}}\right)$$

$$y \geqslant 6. \tag{15}$$

The form of (15) is general, the numerical values within the brackets corresponding to the choice of data from Chang *et al* (1978) and Malozemoff *et al* (1979a, b) for Fe_8B_2; C

is a well defined function which is slowly varying with the variable y. The result of equation (15) is plotted in figure 2 as the continuous curve. The experimental results of Chang *et al* (1978) are plotted as the discontinuous curve for comparison.

4. Conclusions

Calculations are presented for the differential cross section of light backscattered from surfaces of disordered magnetic materials. The configurational average of the cross section is obtained in the site disorder approximation, and the result is shown to be general. It involves novel structure contributions arising due to the magnetic disorder, summed not solely at the centre of the Brillouin zone (as is the case in an ordered system), but over all wavevector excitations. This also contrasts with the cross section for scattering from surfaces of ordered magnetic systems where only k_z summations contribute due to the lack of symmetry along the direction of the z axis. The result is general and distinguishes between the structure contributions arising from the magnetic disorder and surface effects, for all the system one-excitations.

The results are applied in surfaces of amorphous magnets for backscattering from bulk diffusive and magnon-like modes, in the hydrodynamic region of small-wavevector excitations at small frequency shifts. This limit washes out the structure contributions of magnetic disorder, but is nevertheless non-trivial to illustrate the method. Numerical calculations are performed for Fe_8B_2, and compare well with experimental results.

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